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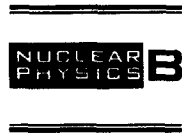
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Super D-branes revisited

E. Bergshoeff^a, P.K. Townsend^{b,1}

^a *Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands*

^b *Institute for Theoretical Physics, University of California at Santa Barbara, CA 93106, USA*

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Abstract

A version of the κ -symmetric super Dp -brane action is presented in which the tension is a dynamical variable, equal to the flux of a p -form world-volume gauge field. The Lagrangian is shown to be invariant under all (super)isometries of the background for appropriate transformations of the world-volume gauge fields, which determine the central charges in the symmetry algebra. We also present the hamiltonian form of the action in a general supergravity background.

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1. Introduction

It is now widely appreciated that super p -branes in a vacuum space-time background are associated with p -form extensions of the standard space-time supersymmetry algebra. In the usual formulation, and for the branes of the ‘old branscan’, this extension arises as a consequence of the non-invariance under space-time supersymmetry transformations of a Wess–Zumino (WZ) term in the world-volume Lagrangian [1]. The p -form is proportional to the tension T and can be considered as a type of ‘classical anomaly’ in which the role of Planck’s constant is taken by T [2]. A simple example is the $D = 11$ supermembrane, in which the WZ term is the pull-back to the world-volume of the 3-form superspace gauge potential of $D = 11$ supergravity [3].

This is not the full story, however. It was shown in [4,5] that the introduction of a p -form gauge potential A allows the construction of an alternative local world-volume

¹ On leave from DAMTP, University of Cambridge, UK.

Lagrangian *without* a WZ term. The new world-volume field does not lead to any additional local degrees of freedom but does lead to an integration constant, which can be identified as the p -brane tension T . When T is non-zero the supersymmetry algebra must include a p -form charge, as before, but since the new Lagrangian is invariant under supersymmetry transformations this charge now has a different origin. To explain this point, let us call the algebra deduced from commutators of supersymmetry transformations acting on world-volume fields the ‘naive’ supersymmetry algebra. This need not be the same as the anticommutator of Noether supercharges because central charges appearing in the latter may commute with all world-volume fields. For example, in the usual formulation of the supermembrane, with WZ term, the world-volume fields are the maps from the world-volume to superspace. The ‘naive’ supersymmetry algebra in this case is therefore the algebra of Killing vector superfields. Assuming a vacuum superspace background, this is just the standard $D = 11$ supersymmetry algebra, which fails to include the 2-form extension implied by the presence of the WZ term.

In the new formulation of the supermembrane there is no WZ term so the naive supersymmetry algebra must include the 2-form extension. What makes this possible is a non-trivial supersymmetry transformation of the 2-form A , which is such that the commutator of two supersymmetry transformations yields a gauge transformation with a parameter that determines the central charge structure. This phenomenon can occur whenever the world-volume fields include gauge fields. It was first noted [6] in the context of the 2-form gauge potential appearing in the $D = 11$ superfivebrane action of [7]. A detailed verification in a very general context, including Type II super D-branes [8–10], has since been provided by Hammer [11]. In this case the ‘naive’ algebra includes a Neveu–Schwarz/Neveu–Schwarz (NS) 1-form charge arising from the presence of the Born–Infeld (BI) gauge field, while all Ramond/Ramond (RR) charges arise from the WZ term.

From the fully non-perturbative point of view there is no real distinction between NS and RR charges, so one might expect there to exist an alternative, manifestly supersymmetric, super D-brane action in which all p -form charges appear already in the ‘naive’ supersymmetry algebra. Here we construct this action via the introduction of a p -form world-volume gauge potential, as suggested by the supermembrane example. This new, manifestly supersymmetric, formulation of the super Dp -brane can be motivated in a number of other ways. It was pointed out in [12] that boundaries (on M5-branes or on ‘M-9-branes’) of open supermembranes could be interpreted as discontinuities in the field strength of a 2-form potential on the $D = 11$ supermembrane world-volume, and that M-theory and superstring dualities would then imply the existence of p -form gauge potentials on the world-volumes of most other M-theory and superstring p -branes, all D-branes in particular. The new super D-string action turns out to be not only manifestly supersymmetric (in a vacuum background) but also manifestly invariant under the $Sl(2; \mathbb{Z})$ duality of IIB superstring theory [13], and it seems to be a general feature that manifest $Sl(2; \mathbb{Z})$ invariance of IIB Dp -brane actions requires the introduction of world-volume p -form gauge potentials [14]. More recently, world-volume p -form gauge potentials have found to be an essential ingredient in the formulation of ‘massive’

p -brane actions [15], i.e. IIA branes in a massive IIA supergravity background.

One aim of this paper is to update our previous work on super D-branes [9] in light of the above discussion. Our results on p -form gauge potentials are complementary to those of [13–15]. We do not consider the implementation of $SI(2; \mathbb{Z})$ invariance in the IIB case, nor massive backgrounds in the IIA case. However, by virtue of these limitations we are able to discuss all D-branes at once, thereby highlighting their common features. The bosonic Lagrangian of the new super D-brane action is simply²

$$L = \frac{1}{2v} [e^{-2\phi} \det(g + \mathcal{F}) + (\star G_{(p+1)})^2], \quad (1.1)$$

where ϕ is the dilaton, v is an independent world-volume density, g is the induced metric, and

$$\mathcal{F} = dV - B \quad (1.2)$$

is the 2-form field strength of the BI 1-form gauge potential V , with B the pullback to the world-volume of the NS 2-form gauge potential. The scalar density $\star G_{(p+1)}$ is the world-volume Hodge dual of a $(p+1)$ -form field strength $G_{(p+1)}$ for the new p -form world-volume gauge potential $A_{(p)}$. In order to deal with all Dp -branes simultaneously it is convenient to combine these p -form gauge potentials into the formal sum³

$$A = \sum_{k=0}^9 A_{(k)}. \quad (1.3)$$

The field strength of A is

$$G = dA - C e^{\mathcal{F}}, \quad (1.4)$$

where C is the formal sum of R–R gauge potentials introduced in [17].

The supersymmetric extension of the Lagrangian (1.1) is formally identical, but with an $N = 2$ superspace replacing the $D = 10$ background space-time. What we have to show is that the Lagrangian so-obtained is κ -symmetric. We do so in the following section.

The super-Poincaré invariance of the super D-brane action in a flat superspace background is a special case of invariance under the (super)isometries of any background allowed by κ -symmetry. Another aim of this paper is to prove this statement. In general, the invariance requires an appropriate choice of transformation rules for world-volume gauge fields. If these include the new p -form gauge field then the Lagrangian (rather than just the action) is invariant. As mentioned above in the context of supertranslations of flat superspace, the transformations of the world-volume gauge fields then determine the central charge structure of the (super)algebra of isometries. Thus, the discussion above generalizes directly to an arbitrary background. This observation is especially important for the D3 brane since it implies that the super D3-brane action in the

² On setting G to zero this reduces to the null D-brane Lagrangian of Ref. [16].

³ This has also been used in Ref. [15].

near-horizon geometry of the D3-brane solution is invariant under the full $SU(2, 2|4)$ isometry of that background. This can be interpreted as a non-linearly realized superconformal symmetry of the super D3-brane action in this background, generalizing the well-known superconformal symmetry of the free $N = 4$ $D = 4$ super-Maxwell action and the more-recently established [18,19] non-linearly realized conformal invariance of the bosonic action.

Finally, we present the hamiltonian formulation of our super D-brane action, generalizing the bosonic results of [20,16] and the flat background IIB results of [21].

2. κ -symmetry

As in Ref. [9], we define $\delta E^A \equiv \delta Z^M E_M^A$, and set $\delta_\kappa E^a = 0$, temporarily leaving open the choice of $\delta_\kappa E^\alpha$. The κ -symmetry variation of the BI field V is

$$\delta_\kappa V_i = E_i^A \delta_\kappa E^\beta B_{\beta A}, \quad (2.1)$$

which is such as to ensure the ‘supercovariance’ of the variation of \mathcal{F} . Using the standard type II $D = 10$ supergravity superspace constraints, summarized in [9], we find that

$$\delta_\kappa \det(g + \mathcal{F}) = -2i \delta_\kappa \tilde{E} \tilde{N}^i E_i, \quad (2.2)$$

where

$$\tilde{N}^i = \det(g + \mathcal{F}) (g + \mathcal{F})^{ij} \mathcal{P}_+ \gamma_j + \det(g - \mathcal{F}) (g - \mathcal{F})^{ij} \mathcal{P}_- \gamma_j. \quad (2.3)$$

Note that, because of the determinant, \tilde{N}^i is non-singular even when $(g \pm \mathcal{F})$ has no inverse. The (reducible) world-volume Dirac matrices γ_i are the pullbacks $E_i^a \Gamma_a$ of the space-time Dirac matrices (more generally, the antisymmetrized product of p world-volume Dirac matrices defines a matrix-valued world-volume p -form $\gamma_{(p)}$ with components $\gamma_{i_1 \dots i_p}$), and

$$\mathcal{P}_\pm = \begin{cases} \frac{1}{2}(1 \pm \Gamma_{11}) & \text{IIA} \\ \frac{1}{2}(1 \pm \sigma_3) & \text{IIB.} \end{cases} \quad (2.4)$$

We find, similarly, that

$$\delta_\kappa [C e^\mathcal{F}] = d(i_{\delta Z} C e^\mathcal{F}) + i \delta_\kappa \tilde{E} \gamma e^\mathcal{F} E, \quad (2.5)$$

where $i_{\delta Z} C$ is the contraction of C with the vector superfield $\delta_\kappa Z^M \partial_M$, and γ is the formal sum

$$\gamma = \begin{cases} e^{-\phi} \sum_{p=0}^8 \gamma_{(p)} (\Gamma_{11})^{(p-2)(p-6)/4} & \text{IIA} \\ e^{-\phi} \sum_{p=1}^9 \gamma_{(p)} \mathcal{P}(\sigma_1)^{(p-3)(p-7)/4} (i\sigma_2)^{(p-1)(p-9)/4} & \text{IIB} \end{cases} \quad (2.6)$$

where \mathcal{P} is the IIB chiral projector on $D = 10$ spinors.

We choose the κ -transformation of A such as to ensure the ‘supercovariance’ of the transformation of the field strength G . This requires

$$\delta A = (i_{\delta Z} C) e^{\mathcal{F}}, \quad (2.7)$$

which leads to

$$\delta_{\kappa} G = -i \delta_{\kappa} \tilde{E} \gamma e^{\mathcal{F}} E. \quad (2.8)$$

For a Dp -brane we must select the $(p+1)$ -form in this formal sum, and then take the world-volume Hodge dual. At this point it is convenient to introduce the matrix ⁴

$$\Xi_{(0)} = \frac{1}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} \gamma_{i_1 \dots i_{p+1}}, \quad (2.9)$$

which satisfies

$$\Xi_{(0)}^2 = (-1)^{p(p+1)/2} \det g. \quad (2.10)$$

We may then write the variation of the world-volume scalar $\star G$ as

$$\delta_{\kappa} \star G = -i e^{-\phi} \delta_{\kappa} \tilde{E} \tilde{M}_{(p)}^i E_i, \quad (2.11)$$

where ⁵

$$\tilde{M}_{(p)}^i = \sum_{n=0} \frac{1}{2^n n!} \gamma^{i j_1 k_1 \dots j_n k_n} \Xi_0 \mathcal{F}_{j_1 k_1} \dots \mathcal{F}_{j_n k_n} \times \begin{cases} (-\Gamma_{11})^{n+(p-2)/2} & \text{IIA} \\ (-\sigma_3)^{n+(p-3)/2} i\sigma_2 & \text{IIB} \end{cases} \quad (2.12)$$

We may now put together the above results to find the κ -variation of the proposed new super D-brane Lagrangian, at least in bosonic backgrounds. In such backgrounds the dilaton has vanishing κ -variation, so we have

$$\delta_{\kappa} [e^{-2\phi} \det(g + \mathcal{F}) + (\star G)^2] = -2i e^{-2\phi} \delta_{\kappa} \tilde{E} (\tilde{N}^i + e^{\phi} (\star G) \tilde{M}_{(p)}^i) E_i. \quad (2.13)$$

Given the results in Refs. [5,9], we see that the spinor variation $\delta_{\kappa} E$ must take the form

$$\delta_{\kappa} \tilde{E}_{\alpha} = [\tilde{\kappa} (e^{\phi} \star G + \Xi)]_{\alpha}, \quad (2.14)$$

where Ξ is a matrix with the property

$$\Xi^2 = -\det(g + \mathcal{F}). \quad (2.15)$$

⁴ The matrix $\Xi_{(0)}$, which is just $\sqrt{-\det g}$ times the matrix $\Gamma_{(0)}$ of [9], is well-defined even for a degenerate world-volume metric.

⁵ We have absorbed some factors into the definition of this matrix relative to the matrix $M_{(p)}^i$ of Ref. [9], where they instead appear in Eq. (4.13). The subsequent calculations of Ref. [9], to be summarized below, actually refer to a matrix $M_{(p)}^i$ with these factors, as is clear from the definition of the matrix $K_{(p)}^i$ in Ref. [9].

Clearly Ξ must reduce (up to a sign) to Ξ_0 when \mathcal{F} vanishes. Again using the results of [9] it follows that

$$\Xi = \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \dots j_n k_n} \mathcal{F}_{j_1 k_1} \dots \mathcal{F}_{j_n k_n} \mathcal{J}_{(p)}^{(n)}, \quad (2.16)$$

where

$$\mathcal{J}_{(p)}^{(n)} = \begin{cases} (\Gamma_{11})^{n+(p-2)/2} \Xi_{(0)} & \text{(IIA)} \\ (-1)^n (\sigma_3)^{n+(p-3)/2} i\sigma_2 \otimes \Xi_{(0)} & \text{(IIB)} \end{cases} \quad (2.17)$$

We now claim that

$$e^{-2\phi} (e^\phi \star G + \Xi) (\tilde{N}^i + e^\phi (\star G) \tilde{M}_{(p)}^i) \equiv \tilde{M}_{(p)}^i [e^{-2\phi} \det(g + \mathcal{F}) + (\star G)^2]. \quad (2.18)$$

The basis for this claim is the calculation sketched in [9] in the course of which it was noted that terms involving a square root of a determinant cancel separately. Thus, the calculation of [9] actually establishes *two* identities, which are

$$\tilde{N}^i + \Xi \tilde{M}_{(p)}^i \equiv 0, \quad \Xi \tilde{N}^i - \det(g + \mathcal{F}) \tilde{M}_{(p)}^i = 0. \quad (2.19)$$

These identities are precisely those needed for (2.18), from which it follows that

$$\delta_\kappa [e^{-2\phi} \det(g + \mathcal{F}) + (\star G)^2] = -2i(\bar{\kappa} \tilde{M}_{(p)}^i E_i) [e^{-2\phi} \det(g + \mathcal{F}) + (\star G)^2]. \quad (2.20)$$

The new super Dp-brane Lagrangian is therefore invariant provided that we choose the κ -variation of the Lagrange multiplier to be

$$\delta_\kappa v = -2iv(\bar{\kappa} \tilde{M}_{(p)}^i E_i). \quad (2.21)$$

We have now established κ -symmetry. The A field equation implies that $\star G$ is a constant. The remaining equations are then those of the standard super D-brane action with the tension equal to the constant $\star G$.

3. Symmetries from background isometries

We shall now consider symmetries of the new super D-brane action arising from isometries of the background. These are generated by Killing vector superfields with respect to which the Lie derivatives of all superspace field strengths vanish. This implies, in particular that the induced metric g_{ij} is invariant. Let $\xi_\alpha = \xi_\alpha^M(Z) \partial_M$ be the set of Killing vector superfields with (anti)commutators

$$[\xi_\beta, \xi_\gamma] = f_{\beta\gamma}^\alpha \xi_\alpha, \quad (3.1)$$

so that $f_{\beta\gamma}{}^\alpha$ are the structure constants of the Lie (super)algebra of isometries. It will prove convenient to introduce the transformations on space-time superfields generated by ξ_α via a BRST operator s . Thus,

$$sZ^M = c^\alpha \xi_\alpha^M, \quad (3.2)$$

where c^α are set of *constant* ghost ‘fields’ with BRST transformation

$$sc^\alpha = \frac{1}{2} c^\gamma c^\beta f_{\beta\gamma}{}^\alpha. \quad (3.3)$$

Note that $s^2 c^\alpha$ is identically zero as a consequence of the Jacobi identity for the structure constants, and that the action of s^2 on Z^M , and hence on all superfields, also vanishes (this being equivalent to $sc^M \equiv 0$).

Since $H = dB$ and $R = dC - CH$ are assumed invariant they are annihilated by s , from which it follows that

$$\begin{aligned} sB &= d\Delta^{(\text{NS})} \\ sC &= d\Delta^{(\text{R})} - \Delta^{(\text{R})}H, \end{aligned} \quad (3.4)$$

where $\Delta^{(\text{NS})}$ is a ghost-valued superspace 1-form and $\Delta^{(\text{R})}$ is a formal sum of ghost-valued superspace forms of all (relevant) degrees. Alternatively, these quantities may be viewed as 1-forms on the isometry group manifold with values in the exterior algebra on superspace. Either way, we see that if the BRST transformations of the world-volume gauge fields V and A are chosen to be

$$sV = \Delta^{(\text{NS})}, \quad sA = e^{\mathcal{F}} \Delta^{(\text{R})}, \quad (3.5)$$

then we have

$$s\mathcal{F} = 0, \quad s\mathcal{G} = 0. \quad (3.6)$$

In other words, *if the superspace background is such that the Lie derivative of each background tensor vanishes then transformations of the independent world-volume gauge potentials V and A can be chosen such that their respective field strengths \mathcal{F} and \mathcal{G} are invariant*, from which it follows that the super D-brane action is invariant.

Having established invariance under background isometries, the next step is to compute the algebra of these symmetry transformations. Since the Lagrangian is invariant, and not just the action, this is equivalent to a computation of the algebra of Noether charges in the Hamiltonian formulation. Since B and C are superfields we have the identities $s^2 B \equiv 0$ and $s^2 C \equiv 0$, which imply that

$$d\mathcal{A}^{(\text{NS})} = 0, \quad d\mathcal{A}^{(\text{R})} = 0, \quad (3.7)$$

where

$$\mathcal{A}^{(\text{NS})} = s\Delta^{(\text{NS})}, \quad \mathcal{A}^{(\text{R})} = e^{\mathcal{F}} s\Delta^{(\text{R})}. \quad (3.8)$$

When these closed superspace forms are exact the transformations of V and A can be removed by gauge transformations and are therefore trivial. Thus $\mathcal{A}^{(\text{NS})}$ and $\mathcal{A}^{(\text{R})}$ may

be viewed as 2-forms on the isometry group manifold with values in cohomology classes of superspace.

Because V and A are not pullbacks of superspace forms they need not be annihilated by s^2 . In fact,

$$s^2 V = \mathcal{A}^{(\text{NS})}, \quad s^2 A = \mathcal{A}^{(\text{R})}. \quad (3.9)$$

Note that the closure of $\mathcal{A}^{(\text{NS})}$ and $\mathcal{A}^{(\text{R})}$ ensures that s^2 annihilates \mathcal{F} and \mathcal{G} (as is, of course, guaranteed by the construction). The closed forms defined by $s^2 V$ and $s^2 A$ determine the topological charges appearing as central charges in the algebra of isometries. They may be calculated explicitly for any particular background. An equivalent explicit calculation for a flat superspace background has been carried out in [11].

4. The super D-brane hamiltonian

In passing to the phase-space form of the D-brane action it is useful to first consider the null super D-brane, for which the Lagrangian is [16]

$$L = \frac{1}{2\nu} e^{-2\phi} \det(g_{ij} + \mathcal{F}_{ij}). \quad (4.1)$$

This is obtained by setting $G = 0$ in (1.1). We can rewrite this as

$$L = \frac{1}{2\nu} e^{-2\phi} \det(\underline{g}_{\underline{ab}} + \underline{\mathcal{F}}_{\underline{ab}}) \{g_{tt} - (g_{t\underline{a}} + \mathcal{F}_{t\underline{a}}) [(g + \mathcal{F})^{-1}]^{\underline{ab}} (g_{t\underline{b}} + \mathcal{F}_{t\underline{b}})\}, \quad (4.2)$$

where we have set $i = (t, \underline{a}_1, \underline{a}_2, \dots, \underline{a}_p)$, i.e. an underlined lower-case Latin index is a ‘worldspace’ index. The matrix $(g + \mathcal{F})^{-1}$, with only worldspace components, is the inverse of the matrix with (only worldspace) components $(\underline{g}_{\underline{ab}} + \underline{\mathcal{F}}_{\underline{ab}})$. This Lagrangian can be further rewritten as

$$L = \frac{1}{2\nu} e^{-2\phi} \det(\underline{g}_{\underline{ab}} + \underline{\mathcal{F}}_{\underline{ab}}) [K_{tt} - K_{t\underline{a}} (K^{-1})^{\underline{ab}} K_{t\underline{b}}], \quad (4.3)$$

where

$$K_{ij} = g_{ij} + \mathcal{F}_{i\underline{a}} (g^{-1})^{\underline{ab}} \mathcal{F}_{j\underline{b}}. \quad (4.4)$$

Here, both K^{-1} and g^{-1} are to be understood as being the inverses of the matrices with only worldspace components, i.e. of $\underline{K}_{\underline{ab}}$ and $\underline{g}_{\underline{ab}}$, respectively. If we now define

$$\lambda = \nu e^{2\phi} / \det(\underline{g}_{\underline{ab}} + \underline{\mathcal{F}}_{\underline{ab}}), \quad (4.5)$$

then we have

$$L = \frac{1}{2\lambda} [K_{tt} - K_{t\underline{a}} (K^{-1})^{\underline{ab}} K_{t\underline{b}}]. \quad (4.6)$$

An equivalent Lagrangian is

$$L = \tilde{P} \cdot \Pi_t + \tilde{E}^a \mathcal{F}_{t\underline{a}} - s^a (\tilde{P} \cdot \Pi_{\underline{a}} + \tilde{E}^b \mathcal{F}_{\underline{ab}}) - \frac{1}{2} \lambda (\tilde{P}^2 + \tilde{E}^a \tilde{E}^b \underline{g}_{\underline{ab}}), \quad (4.7)$$

where

$$(\Pi_i)^a \equiv E_i^a. \quad (4.8)$$

To establish the equivalence one first eliminates the new auxiliary variables \tilde{P}_a and \tilde{E}^a to obtain the Lagrangian

$$L = \frac{1}{2\lambda} [K_{tt} - 2s^a K_{ta} + s^a s^b K_{ab}]. \quad (4.9)$$

Elimination of s^a then yields (4.6).

The Lagrangian (4.7) is a convenient ‘half-way house’ on the way to the true phase-space form of the null D-brane action (the variable \tilde{P} is not the momentum conjugate to X , for example, although it is closely related to it). However, rather than proceed with the null D-brane it is convenient to now pass to the full D-brane action. We first note that the world-volume $(p+1)$ -form $G_{(p+1)}$ in the formal sum G can be written as

$$G_{(p+1)} = dt (G_t)_{(p)}, \quad (4.10)$$

where (G_t) is a p -form on the p -dimensional worldspace. It is the p -form in the formal sum

$$G_t = \dot{A} - dA_t - \dot{Z}^M C_M e^{\mathcal{F}} - \mathcal{F}_t (C e^{\mathcal{F}}), \quad (4.11)$$

where A is now a formal sum of *worldspace* forms, as is A_t , and d is now to be understood as an exterior derivative on worldspace. Similarly, C_M is the restriction to worldspace of the formal sum $i_M C$, where i_M denotes contraction with the vector superfield $\partial/\partial Z^M$. Finally, $\mathcal{F}_t = d\sigma^a \mathcal{F}_{ta}$ is a worldspace 1-form. Specifically,

$$\mathcal{F}_t = \dot{V} - dV_t - \dot{Z}^M B_M, \quad (4.12)$$

where B_M is the restriction to worldspace of the 1-form $i_M B$, so that

$$G_t = \dot{A} - dA_t - \dot{V} (C e^{\mathcal{F}}) - \dot{Z}^M [C_M e^{\mathcal{F}} - B_M (C e^{\mathcal{F}})] + (dV_t) (C e^{\mathcal{F}}). \quad (4.13)$$

Using these results we can now write down a ‘half-way house’ version of the *full* super Dp-brane Lagrangian (1.1), involving G_t . This is

$$L = \tilde{P} \cdot \Pi_t + \tilde{E}^a \mathcal{F}_{ta} + T * (G_t)_{(p)} - s^a (\tilde{P} \cdot \Pi_a + \tilde{E}^b \mathcal{F}_{ab}) - \frac{1}{2} \lambda [\tilde{P}^2 + \tilde{E}^a \tilde{E}^b g_{ab} + T^2 e^{-2\phi} \det(g_{ab} + \mathcal{F}_{ab})], \quad (4.14)$$

where $*$ indicates the *worldspace* Hodge dual. The equivalence can be established as before by elimination of the variables \tilde{P} , \tilde{E}^a and T followed by the redefinition (4.5) and elimination of s^a : the Lagrangian (1.1) is then recovered.

To obtain this action in canonical form we proceed as follows. Omitting total derivatives, and defining the $(p-1)$ -form

$$U_{(p-1)} = (A_t)_{(p-1)} + (-1)^p V_t (C e^{\mathcal{F}})_{(p-1)}, \quad (4.15)$$

we have

$$T(G_t)_{(p)} = \dot{A}_{(p)} T + U_{(p-1)} dT - \dot{V} T (C e^{\mathcal{F}})_{(p-1)} - \dot{Z}^M T [C_M e^{\mathcal{F}} - B_M (C e^{\mathcal{F}})]_{(p)} + (-1)^p V_t T (R e^{\mathcal{F}})_{(p)}, \quad (4.16)$$

where $R = dC - CH$ is the field strength for C ⁶. Thus

$$T * (G_t)_{(p)} = \dot{\phi} T + \phi^{\underline{a}} \partial_{\underline{a}} T - T \dot{V}_{\underline{a}} C^{\underline{a}} - T \dot{Z}^M [C_M - (B_M)_{\underline{a}} C^{\underline{a}}] + (-1)^p T V_t \mathcal{R}, \quad (4.17)$$

where we have defined

$$\phi = *A_{(p)}, \quad \phi^{\underline{a}} = (*U_{(p-1)})^{\underline{a}}, \quad (4.18)$$

and

$$C^{\underline{a}} = [* (C e^{\mathcal{F}})_{(p-1)}]^{\underline{a}}, \quad C_M = *(C_M e^{\mathcal{F}})_{(p)}, \quad \mathcal{R} = *(R e^{\mathcal{F}})_{(p)}. \quad (4.19)$$

Thus (4.14) can now be rewritten as

$$L = \dot{Z}^M P_M + \dot{V}_{\underline{a}} E^{\underline{a}} + \dot{\phi} T - H, \quad (4.20)$$

where the hamiltonian H is a sum of constraints imposed by Lagrange multipliers, and

$$P_M = E_M^{\underline{a}} \tilde{P}_{\underline{a}} - \tilde{E}^{\underline{a}} (B_M)_{\underline{a}} - T [C_M - (B_M)_{\underline{a}} C^{\underline{a}}], \quad (4.21)$$

$$E^{\underline{a}} = \tilde{E}^{\underline{a}} - T C^{\underline{a}}.$$

These equations imply that

$$\tilde{P}_{\underline{a}} = E_{\underline{a}}^M (P_M + E^{\underline{a}} (B_M)_{\underline{a}} + T C_M), \quad (4.22)$$

$$\tilde{E}^{\underline{a}} = E^{\underline{a}} + T C^{\underline{a}}.$$

Since E_{μ}^{α} is invertible, the remaining information contained in (4.22) is the constraint

$$E_{\alpha}^M (P_M + E^{\underline{a}} (B_M)_{\underline{a}} + T C_M) = 0, \quad (4.23)$$

which can be incorporated as a constraint in H imposed by a new fermionic Lagrange multiplier ζ . Thus

$$H = \phi^{\underline{a}} T_{\underline{a}} + V_t \mathcal{G} + s^{\underline{a}} \mathcal{H}_{\underline{a}} + \lambda \mathcal{H} + \zeta^{\alpha} S_{\alpha}, \quad (4.24)$$

where

$$T_{\underline{a}} = -\partial_{\underline{a}} T, \quad (4.25)$$

$$\mathcal{G} = -\partial_{\underline{a}} \tilde{E}^{\underline{a}} + (-1)^{p+1} T \mathcal{R},$$

$$\mathcal{H}_{\underline{a}} = \tilde{P} \cdot \Pi_{\underline{a}} + \tilde{E}^{\underline{b}} \mathcal{F}_{\underline{a}\underline{b}},$$

$$\mathcal{H} = \frac{1}{2} [\tilde{P}^2 + \tilde{E}^{\underline{a}} \tilde{E}^{\underline{b}} g_{\underline{a}\underline{b}} + T^2 e^{-2\phi} \det(g + \mathcal{F})],$$

$$S_{\alpha} = E_{\alpha}^M [P_M + E^{\underline{a}} (B_M)_{\underline{a}} + T C_M],$$

⁶ As before we use the standard superspace convention that exterior differentiation ‘starts from the right’.

with \tilde{P}_a and \tilde{E}^a given by (4.22). The constraint functions \mathcal{T}_a , \mathcal{G} , \mathcal{H}_a and \mathcal{H} are ‘first class’ (in Dirac’s terminology) and generate the p -form gauge transformations, BI gauge transformations, worldspace diffeomorphisms and time translations, respectively. The fermionic constraint functions \mathcal{S} are half first class and half second class; the first class constraints generate κ -symmetry transformations.

5. Discussion

In this paper we have presented a new formulation of the super Dp -brane action in which the tension appears as an integration constant in the equation of motion of a new p -form gauge potential. In this form of the action the full centrally extended supertranslation algebra is realized as the ‘naive’ algebra of transformations of world-volume fields. In the standard form, only the NS charges are ‘naive’ while the RR charges arise from the presence of the WZ term. In the zero tension limit the RR charges vanish and one is left with the ‘naive’ algebra of the null super D-brane. The latter does not coincide with the standard supersymmetry algebra (in contrast to, for example, the null $D = 11$ supermembrane) because the BI field does not decouple in the null limit.

The results just summarized provide a clear understanding of the origin of the various p -form charges in the supertranslation algebra of the super Dp -brane. One of the original motivations for this work was to obtain a similar understanding of the recently determined central charge structure of the M5-brane [6]. In that case the WZ term was found to be responsible for the full 5-form charge but for only half of the 2-form charge. The remaining half is explained by the fact that, because of the world-volume 2-form gauge potential, the ‘naive’ supersymmetry algebra differs from the standard one. The D-brane results reported here are similar but with the additional feature that one can exhibit the ‘naive’ algebra as the algebra in the null limit, thereby isolating the naive (NS) and WZ (RR) contributions. In the M5-brane case there is apparently no way to achieve this separation: our attempts to define a null limit of the M5-brane (in the ‘temporal gauge’) simply led to the standard null super 5-brane in which the 2-form potential is absent.⁷ In view of this, one might try instead to unify the contributions to the central charge structure of the M5-brane supersymmetry algebra by realizing the full algebra as the naive supersymmetry algebra on world-volume fields, as is achieved for D-branes via the new scale-invariant action presented here. It is not yet clear to us whether this is possible. As pointed out in [12], a p -form world-volume gauge potential is natural for objects that may, like D-branes, have boundaries on other branes, but the M5-brane is always closed. A 5-form gauge potential has been successfully introduced in a recent reformulation of the M5-brane action [22], but as the self-duality constraint is incorporated at the level of the field equations this action could not be used to ex-

⁷ There is no conflict with supersymmetry here because, as pointed out in [5], space-time supersymmetry and κ supersymmetry do *not* imply world-volume supersymmetry for null branes.

tract the central charge structure in the manner envisaged here. We shall return to the M5-brane in a future publication [23].

We have also shown that the new super D-brane Lagrangian is invariant under all isometries of the supergravity background, provided that the world-volume gauge fields are taken to transform appropriately; this implies that the ‘old’ super D-brane Lagrangian is invariant up to a total derivative. One advantage of the new Lagrangian is that its invariance means that the full symmetry algebra, with any possible central extensions, may be deduced from the algebra of transformations of the world-volume fields. Central extensions of the algebra of Killing vector superfields are coded in the action of a BRST operator s on the world-volume gauge fields: one finds that s^2 generates gauge transformations of these fields with parameters determined by closed superspace forms.

A case of current interest to which this analysis is applicable is the super D3-brane in the maximally supersymmetric $\text{adS}_5 \times S^5$ IIB background, which has isometry supergroup $SU(2, 2|4)$. An appropriate embedding of a super D3-brane world-volume in this background has been shown to result in an interacting world-volume field theory in which the $SU(2, 2)$ subgroup acts as a non-linearly realized conformal symmetry on the bosonic fields [19]. The results obtained here show that the full action is invariant under the full $SU(2, 2|4)$ isometry supergroup, which can be interpreted as a non-linearly realized superconformal symmetry. More precisely, we have shown that the action is invariant under transformations that close to $SU(2, 2|4)$ on the world-volume scalar (and, implicitly, spinor) fields. Closure on the world-volume gauge fields might, in principle, require the introduction of additional charges that are central with respect to the linearly realized subgroup of $SU(2, 2|4)$, which is $(3 + 1)$ -dimensional super-Poincaré and its $SU(4)$ group of automorphisms. Specifically, one might expect the D3-brane to be associated with a 3-form charge in the adS superalgebra. From the perspective of adS_5 the space components of this 3-form (which are naturally associated with a 3-brane) would be dual to a time component of a dual 2-form, but there is already a charge of this type in the adS algebra: it is the generator of boosts in the space direction orthogonal to the 3-brane. That this must be the 3-brane charge can be seen from the fact that the adS_5 space-time can itself be viewed as a p -brane [24], for which the only candidate charge is the one already in the adS algebra. There are therefore no *new* charges in the symmetry algebra relative to the algebra of isometries, so that the symmetry group of the super D3-brane is just the superconformal group $SU(2, 2|4)$.

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